ERGODIC AVERAGES AND HELICAL TRANSFORMS

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Abstract. Our setting is a probability space $(X, \mathcal{B}, \mu)$, equipped with an invertible, ergodic, measure-preserving transformation $T : X \to X$. Most ergodic theorists learn quite early and remember until quite late that

$$\lim_{n\to\infty} \frac{1}{n} \sum_{j=1}^{n} f(T^j x) = \int_X f d\mu,$$

both almost surely and in L^2, for each $f \in L^2$. With almost no extra effort, one sees that the averages

$$\frac{1}{n} \sum_{j=1}^{n} e^{(ij\theta)} f(T^j x)\,, \quad \theta \in [-\pi, \pi]$$

satisfy: for each fixed θ, they converge (as n tends to infinity) for almost every x. Wiener and Wintner showed that the stronger statement - for almost every x, the averages converge for all θ - holds.

We discuss analagous and related results for the averages

$$\sum_{|j|=1}^{n} \frac{f(T^j x)}{j}\,, \quad \text{and} \quad \sum_{|j|=1}^{n} \frac{e^{(ij\theta)} f(T^j x)}{j}$$

and show how these averages arise naturally in the context of operator theory and Fourier series.

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