2020 Coast Combinatorics Conference (32nd)
7-8 March, 2020
University of Victoria
Cornett Building, Room A121

Schedule and Abstracts

Saturday, March 7, Cornett A121

Boardwalk Cafe (University Centre) opens at 9:00 and BiblioCafe (Library) opens at 9:30

10:00 *Source-Sink Diffusion*
Danielle Cox, Mt. St. Vincent University

10:30 *Serial Exchanges of Bases in Matroids*
Sean McGuinness, Thompson Rivers University

11:00 *A New Representation for Mutually Orthogonal Frequency Squares*
Tabriz Popatia, Simon Fraser University

11:30 **Lunch break**: options nearby

1:00 IdeaFest Event, *How to make a really irritating Sudoku puzzle*
Peter Dukes, University of Victoria, Canada. **David Strong Building C118**.

2:00 *Pigeonhole classes of matroids*
Dillon Mahew, Victoria University, New Zealand

2:30 *Fault tolerance in cryptography using cover-free families*
Thais Bardini Idalino, Simon Fraser University

3:00 *On the Strong Nine Dragon Tree Conjecture*
Benjamin Moore, University of Waterloo

3:30 **Coffee break**. Coffee and snacks served

4:00 *Categorifications of algebras via towers of groups*
Farid Aliniaeifard, University of British Columbia - Vancouver

4:30 *The Oriented Chromatic Number of Cubic Graphs*
Chris Duffy, University of Saskatchewan

5:00 **End** of Saturday's talks.

7:00 Optional informal dinner at Romeo's Place, 1703 Blanshard St., `www.romeos.ca`

Sunday, March 8, Cornett A121

Boardwalk Cafe (University Centre) opens at 9:00 and BiblioCafe (Library) opens at 9:30

10:00 *Matroids on graphs: One big happy family*
Darryl Funk, Douglas College

10:30 *A positivity phenomenon in Elser's Gaussian-cluster percolation model*
Hays Whitlatch, Gonzaga University

11:00 *Intersection distribution, non-hitting index, and their applications*
Shuxing Li, Simon Fraser University

11:30 *On a Conjecture of Nagy on Extremal Densities*
Amites Sarkar, Western Washington University

noon **Conference Ends**

Abstracts in alphabetical order by speaker's surname

Farid Aliniaeifard, University of British Columbia - Vancouver
Categorifications of algebras via towers of groups

There is a long tradition of categorifying combinatorial Hopf algebras by the modules of a tower of algebras (or even better via the representation theory of a tower of groups). From the point of view of combinatorics, such a categorification supplies canonical bases, inner products, and a natural avenue to prove positivity results. Recent ideas in supercharacter theory have made fashioning the representation theory of a tower of groups into a Hopf structure more tractable. As a demonstration, this talk reports on the results of the following challenge: (1) Pick a well-known combinatorial Hopf algebra, (2) Find a way to categorify the structure via a tower of groups. This is joint work with Nat Thiem.

Danielle Cox, Mt. St. Vincent University
Source-Sink Diffusion

In this talk we will introduce the diffusion process on graphs with the addition of sources and sinks. In particular, we will provide results regarding the periodicity of the process. This is joint work with Todd Mullen (Dalhousie University), Emily Wright (MSVU) and Jesse Preston (MSVU).

Chris Duffy, University of Saskatchewan
The Oriented Chromatic Number of Cubic Graphs

The definition of homomorphism of oriented graphs gives rise to a definition of proper colouring that takes into account the orientation of the arcs. Despite the attention that this notion of colouring has received in the literature over the last twenty years, sharp bounds on the chromatic number of some fundamental oriented graph classes remain elusive. In 1997 Sopena conjectured that every orientation of a connected cubic graph admits a 7-colouring. In this talk we will examine the current state of Sopena's conjecture, noting interplay in techniques and results between this and related problems.

Peter Dukes, University of Victoria, Canada
How to make a really irritating Sudoku puzzle

This talk is part of UVic IdeaFest and takes place in David Strong Building room C118.

Do you strive to complete the Sudoku puzzle in the paper? Or are you distracted by the geometric patterns of the tiled floor? Learn how the two are more related than it seems and how new research can help determine what makes a puzzle solvable or impossible in a presentation and discussion session. Then enjoy a hands-on exploration of the research by modelling and solving some puzzles for yourself. Other presenters: Brittany Halverson-Duncan, Kieka Mynhardt, Joanna Niezen and Coen del Valle.

Darryl Funk, Douglas College
Matroids on graphs: one big happy family

Matroids represented by graphs have come to play an important role in the matroid structure theory of Jim Geelen, Burt Gerards, & Geoff Whittle. These are the classes of frame matroids and single-element lifts of graphic matroids. Geelen, Gerards, & Whittle have recently introduced another class of matroids represented by graphs: "quasi-graphic" matroids. This class is a common generalisation of these other two graph-like classes. We describe a unified method for representing matroids in these classes with graphs.

Thais Bardini Idalino, Simon Fraser University
Fault tolerance in cryptography using cover-free families

We explore solutions for fault tolerance in cryptography using techniques of identification of defective elements used in nonadaptive combinatorial group testing. More specifically, we use the well studied cover-free families (CFFs). A d-cover-free family d-CFF(t, n) is a set system with n subsets of a t-set, where the union of any d subsets does not contain any other. A d-CFF(t, n) allows for the identification of up to d defective elements in a set of n elements by performing only t tests (typically $t << n$).

We explore different aspects of cover-free families in order to better approach fault tolerance problems. For instance, while CFFs are used as a solution to many problems in cryptography, we note that some of those problems require CFFs with increasing n. In this context, we investigate monotone, nested, and embedding sequences of CFFs, and propose constructions using techniques from combinatorial design theory and finite fields. Some of the proposed constructions achieve the best possible compression ratio, which meets the best known upper bound. Monotone, nested and embedding sequences of CFFs can also be applied in any group testing problem that is dynamic in nature.

[1] IDALINO, T. B.; MOURA, L., *Efficient Unbounded Fault-Tolerant Aggregate Signatures Using Nested Cover-Free Families.* In: International Workshop on Combinatorial Algorithms, IWOCA 2018. Lecture Notes in Computer Science, vol 10979, pages 52?64. Springer, Cham.

[2] IDALINO, T. B.; MOURA, L., *Embedding cover-free families and cryptographical applications.* Advances in Mathematics of Communications, 13 (2019), 629?643.

Shuxing Li, Simon Fraser University
Intersection distribution, non-hitting index, and their applications

For a point set of the classical projective plane PG$(2, q)$, we introduce the concept of intersection distribution, which reflects how this point set interacts the lines of PG$(2, q)$. Among the intersection distribution, a particularly interesting quantity is the so called non-hitting index, which equals the number of lines not intersecting with the point set. We consider the case where the point set has $q + 1$ points, so that a compact polynomial representation of the point set is usually available, and have the following results:

1. Employing a geometric approach, we characterize the point sets whose non-hitting indices are close to the lower or upper bounds.
2. Employ an algebraic approach, we compute the intersection distributions of several families of

point sets, which can be represented by monomials over finite fields.
3. Using the information of intersection distributions, we construct several infinite families of Kakeya sets in affine planes with prescribed sizes.

This is joint work with Alexander Pott.

Dillon Mahew, Victoria University, New Zealand
Pigeonhole classes of matroids

Matroids are abstractions of finite configurations of vectors. If V is a vector space over the field F, then any finite subset of V gives rise to a matroid, and this matroid is said to be F-representable.

General classes of matroids are very abstract, and exhibit wild behaviour. But if F is a finite field, then the class of F-representable matroids behaves in many ways like a minor-closed class of graphs. In particular, Hlineny proved that any sentence in the monadic second-order logic of matroids can be tested in polynomial time, as long as we restrict the input to F-representable matroids with bounded branch-width. This is a matroid analogue of Courcelle's Theorem for graphs with bounded tree-width.

Our work generalises Hlineny's Theorem by identifying the structural properties that drive its engine. Classes that have the so-called pigeonhole property resemble finite-field-representable matroids in some important ways, and we can extend Hlineny's Theorem to these pigeonhole classes. Using these ideas, we have been able to show that Hlineny's Theorem holds for several other natural classes of matroids.

This is joint work with Daryl Funk, Mike Newman, and Geoff Whittle.

Sean McGuinness, Thompson Rivers University
Serial Exchanges of Bases in Matroids

Suppose B_1 and B_2 are two disjoint bases in a matroid M. For subsets $X \subseteq B_1$ and $Y \subseteq B_2$, we say that X is *serially exchangeable* with Y if there exist orderings $x_1 \preceq x_2 \preceq \cdots \preceq x_k$ and $x_1 \preceq x_2 \preceq \cdots \preceq x_k$ of X and Y, respectively, such that for $i = 1, 2, \ldots, k$, $B_1 - x_1 - x_2 - \cdots - x_i + y_1 + y_2 + \cdots + y_i$ and $B_2 - y_1 - y_2 - \cdots - y_i + x_1 + x_2 + \cdots + x_i$ are bases. Kotlar and Ziv conjectured that for any $X \subseteq B_1$ there exists $Y \subseteq B_2$ for which X is serially exchangeable with Y. Let $\mathcal{M}_q$ be the set of matroids representable over $\mathrm{GF}(q)$. We show that if the above conjecture holds for all matroids $M \in \mathcal{M}_q$ of rank at most $2q^{2k}$, then it holds for all matroids in $\mathcal{M}_q$.

Benjamin Moore, University of Waterloo
On the Strong Nine Dragon Tree Conjecture

A well known theorem of Nash-Williams asserts that a graph decomposes into k forests if and only if the fractional arboricity of the graph is at most k, and a similar theorem of Hakimi asserts that a graph decomposes into k pseudoforests if and only if the maximum average degree of the graph is at most $2k$. I'll discuss generalizations of these theorems. Namely, that a graph with maximum average degree at most $2k + d/(k + d + 1)$ decomposes into $k + 1$ pseudoforests such that one of the pseudoforests has each connected component containing at most d edges, and that a graph with fractional arboricity at most $k + d/(k + d + 1)$ decomposes into $k + 1$ forests such that one of the forests has every connected component containing at most $f(k,d)$ edges, for a particular function $f(k,d)$. This is joint work with Logan Grout.

Tabriz Popatia, Simon Fraser University
A New Representation for Mutually Orthogonal Frequency Squares

Mutually orthogonal frequency squares (MOFS) of type $F(m\lambda;\lambda)$ generalize the structure of mutually orthogonal Latin squares, with m elements each appearing exactly λ times in every row and in every column. Although complete sets of MOFS have been constructed from various combinatorial designs, less is known about maximal incomplete sets of MOFS. Previous study of maximal incomplete sets of MOFS depends critically on the size m of the symbol set being 2. We introduce a new method of analysis for MOFS with arbitrary symbol set size m, using a representation as a linear combination of $\{0,1\}$ indicator squares. We use this representation to simplify and extend previous analysis of incomplete sets of MOFS for all values of $m > 2$. This leads to a new direct elementary proof of the maximum size of a set of MOFS, and a new criterion for a set of incomplete MOFS to be maximal.

Amites Sarkar, Western Washington University
On a Conjecture of Nagy on Extremal Densities

Given two graphs G and H, let $N(G,H)$ be the number of copies of G in H. Fix a graph G and some $b \in (0,1)$. Among all graphs H on n vertices, with edge density at most b, how large can $N(G,H)$ be? Nagy conjectured that, as n grows, $N(G,H)$ is always asymptotically maximized by either a quasi-clique or a quasi-star.

Results by a large number of authors, including Ahlswede and Katona, and Alon, seemed to support this conjecture, and it was known to be true in a large number of cases. In this talk, we show that the conjecture is in fact false for infinitely many graphs G. We also present a new conjecture, and discuss the evidence supporting it.

This talk is based on joint work with Nick Day.

Hays Whitlatch. Gonzaga University
A positivity phenomenon in Elser's Gaussian-cluster percolation model

Veit Elser proposed a random graph model for percolation in which physical dimension appears as a parameter. Studying this model combinatorially leads naturally to the consideration of numerical graph invariants which we call *Elser numbers* $\mathrm{els}_k(G)$, where G is a connected graph and k a nonnegative integer. Elser had proven that $\mathrm{els}_1(G) = 0$ for all G. By interpreting the Elser numbers as Euler characteristics of appropriate simplicial complexes called *nucleus complexes*, we prove that for all graphs G, they are nonpositive when $k = 0$ and nonnegative for $k \geq 2$. The last result confirms a conjecture of Elser.