UVic Mathematics Competition 2011

September 20th 2011

No calculators allowed. All four questions carry equal credit. Partial answers will earn little or no credit

1. Suppose that two twins are playing with a collection of $4n$ bricks, of which half are red and half are blue. Suppose also that of the bricks, one half are made of wood and one half are made of plastic.

   Is it always possible for the twins to share the bricks so that each twin's pile contains n red bricks and n blue bricks; and also each twin's pile contains n plastic bricks and n wooden bricks?

2. Let $f(x) = \sum_{n=1}^{k} a_n \sin(b_n x)$, where the a_n are real and the b_n are distinct and positive. Show that there exists a real number x such that $f(x) > \sqrt{\sum a_n^2/2}$.

3. Prove that
$$\sum_{n=2011}^{6011} \sqrt{n^2+1}$$
   is not an integer.

4. Find the largest n for which there exist distinct points $P_1, P_2, \ldots, P_n$ in the plane and real numbers $r_1, r_2, \ldots, r_n$ such that the distance between P_i and P_j equals $r_i + r_j$ for any $i \neq j$.