

COURSE OUTLINE MATH 311: Linear Algebra

Instructors

| Dr. Christopher Eagle | Lecture Section: A01 |
|-----------------------------|---|
| Email: eaglec@uvic.ca | Office: DTB A441 |
| Research Interests: Mathema | atical logic, the foundations of mathematics, functional analysis |

General Course Information

Number of Units 1.5

Pre-requisites (One of MATH 110, MATH 133, MATH 211, MATH 233A) and (MATH 212 or MATH 233C).

Office Hours and Assistance

See CourseSpaces for office hours times and days. Note that these might change during the term.

Math Club Students in Undergraduate Mathematics and Statistics (SUMS) was founded in 2014 as the reincarnation of a previous undergraduate course union that had been inactive for a few years. Please see http://www.uvic.ca/science/math-statistics/ current-students/undergraduate/sums/index.php for more information.

Learning Objectives

This course is a theoretical, proof-based treatment of vector spaces over fields and the linear functions between them.

A major theme in much of mathematics, and especially algebra, is *classification*. Classifying the objects of some kind generally involves describing the appropriate notion of "the same" for those objects, and then producing a list of objects with the property that each object of the type we are classifying is "the same" as exactly one object on the list. In many contexts classification is very difficult, or even impossible, but linear algebra offers a setting where many useful classifications are within reach. In this course we will carry out several classifications, and along the way we will develop the theory of linear algebra in a more abstract setting than you saw in Math 211. The material we will cover corresponds to chapters 1-8 of the required textbook.

The notion of a vector space gives a general setting in which to develop linear algebra. The subspaces of \mathbb{R}^n that you worked with in Math 211 are examples of vector spaces over \mathbb{R} , but there are many other interesting vector spaces, and we will also consider vector spaces over fields other than \mathbb{R} . Our first major classification result will be, for each field \mathbb{F} , a



classification all finite-dimensional vector spaces over \mathbb{F} up to isomorphism. The material needed here is chapters 1-3 of the textbook.

Most of the content of linear algebra concerns not just vector spaces, but the linear functions between them. We will study the general properties of linear functions, but then we will focus on the case of a linear function from a vector space to itself. The second classification theorem we will prove classifies all linear functions from a finite dimensional complex vector space to itself up to similarity. This classification is based on eigenvalues and eigenvectors, and generalizes the method of diagonalization that you learned in Math 211. This part of the course corresponds to chapters 4, 5, and 8 of the textbook.

In general, vector spaces do not have enough structure to define the geometric notions of length and angle. Inner product spaces, which are vector spaces with an additional operation that generalizes the dot product, are the right setting for these notions. We will develop the theory of inner product spaces (primarily over the complex numbers). Our last classification result will classify, up to unitary conjugacy, all normal linear functions from a complex inner product space to itself. These results are in chapters 6 and 7 of the textbook.

The specific topics to be covered include: Abstract vector spaces, subspaces, and direct sums; span, linear independence, bases, and dimension; linear functions, matrix representations of linear functions, invertibility, isomorphism of vector spaces; eigenvalues and eigenvectors, diagonalizability, Jordan canonical form; inner product spaces, self-adjoint and normal linear operators, the Spectral Theorem.

Course Material and Online Resources

- **Textbook** Linear algebra done right, 3rd edition, by Sheldon Axler. Our library has a digital version available for free. Printed copies of the book may be purchased at the UVic Bookstore.
- **Course webpage** Course materials, including homework assignments and this outline, will be available through CourseSpaces.
- Calculator The use of a calculator is *not* permitted on the tests or exams in this course.

Class Meetings

Lectures Lectures will be held on Mondays, Wednesdays, and Thursdays, 3:30-4:20pm, in Maclaurin D114. The first class is on Wednesday, January 3.

Evaluation and Grading

Your final percentage grade will be computed according to the following scheme.



| Item | Date(s) | Weight |
|----------------------|-----------------------------------|--------|
| Homework Assignments | See below | 30% |
| Term tests | February 1 and March 15, in class | 25% |
| Final exam | TBA | 45% |

- Grading Percentage scores will be converted to letter grades according to the universitywide standard table (http://web.uvic.ca/calendar2018-01/undergrad/info/regulations/ grading.html).
- **Assignments** Take-home problem sets will published on CourseSpaces on alternate Wednesdays, and due two weeks later. We will also have some in-class assignments - for the first part of the term these will occur near the end of class on Mondays. The schedule for in-class assignments is subject to change, and I will announce any such changes in class.

On the take-home problem sets you are encouraged to talk to me, or to each other. However, the final work that you submit must be your own. A useful guide is that you should write your final solutions on your own, and the ideas should be expressed in your own words. If you are unsure about how much collaboration is permitted or appropriate, please speak to me early in the term.

For the in-class assignments you may get help from me, other students, the textbook, or your notes - the only thing I ask you not to do during the in-class assignments is make use of electronic resources. Each in-class assignment will be worth 1% of your final grade. The remaining portion of the assignment grade will be allocated to the take-home problem sets.

- **Tests** There will be two term tests, one on Thursday, February 1 and one on Thursday, March 15, during class time. Unlike the in-class assignments, for the tests you will work alone without access to your notes or textbook. I will provide more details about the format and content of the tests closer to the test dates.
- Missing work Tests: If you miss one of the tests for a valid reason (such as illness, family emergency, or religious obligation) then you must provide documentation as soon as possible. In this case the missing grade will be replaced by your grade on the final exam. Missing both term tests will result in an incomplete grade for the course.
 - Assignments: Late assignments will ordinarily not be accepted. If you must miss submitting an assignment due to extended illness or other valid reason, please contact me as soon as possible to make arrangements.
 - In-class assignments: In general, there will be no opportunity to make up missed in-class assignments. If you know in advance that you must miss one of these assignments, or if you miss one due to illness or emergency, please contact me to make arrangements.
- **Unclaimed work** All graded term work in this course will be returned to you during lecture. If you miss the lecture when a graded piece of work is returned, you can collect it during my office hours, or make and appointment with me to pick it up from



my office at another time. Any term work that is not collected by the end of the final examination period will be securely shredded.

- Final Examination Off-schedule final examinations (i.e., deferred examinations) are given only in accordance with the university policy as outlined in the Calendar. If you are unable to write a final examination due to illness, accident or family affliction, please refer to the following webpages for detailed instructions how to proceed: http://web. uvic.ca/calendar2018-01/undergrad/info/regulations/concessions.html and http://web.uvic.ca/calendar2018-01/undergrad/info/regulations/exams.html. Students are strongly advised not to make plans for travel or employment during the final examination period as special arrangements will not be made for examinations that conflict with such plans.
- Supplemental Examinations. The Department of Mathematics and Statistics does not award 'E' grades or offer Supplemental Examinations in any of its courses.
- Accessibility Students with diverse learning styles and needs are welcome in this course. In particular, if you have a disability/health consideration that may require accommodations, please feel free to approach your instructor and/or the Centre for Accessible Learning (CAL) as soon as possible. The CAL staff are available by appointment to assess specific needs, provide referrals and arrange appropriate accommodations http://www.uvic.ca/cal. The sooner you let us know your needs the quicker we can assist you in achieving your learning goals in this course.
- **Commitment to Inclusivity and Diversity** The University of Victoria is committed to promoting, providing and protecting a positive, supportive and safe learning and working environment for all its members.

Departmental Policies and Ethics

(See https://www.uvic.ca/science/math-statistics/current-students/undergraduate/ course-policies/index.php for more information.)

- Attendance The university Calendar states 'Students are expected to attend all classes in which they are enrolled.' (see http://web.uvic.ca/calendar2018-01/undergrad/ info/regulations/attendance.html). Our courses are conducted on that basis. If you miss an announcement (information concerning midterms, corrections to assignment, etc.) because you did not attend class, you must accept the consequences of not having learned of the change.
- **Guidelines on Religious Observances** Where classes or examinations are scheduled on the holy days of a religion, students may notify their instructors, at least two weeks in advance, of their intention to observe the holy day(s) by absenting themselves from classes or examinations. Instructors will provide reasonable opportunities for such students to make up work or missed examinations.
- Academic Integrity Academic integrity is intellectual honesty and responsibility for academic work that you submit individual or group work. It involves commitment to the



values of honesty, trust, and responsibility. It is expected that students will respect these ethical values in all activities related to learning, teaching, research, and service. Therefore, plagiarism and other acts against academic integrity are serious academic offenses.

The responsibility of the institution

Instructors and academic units have the responsibility to ensure that standards of academic honesty are met. By doing so, the institution recognizes students for their hard work and assures them that other students do not have an unfair advantage through cheating on essays, exams, and projects.

The responsibility of the student

Plagiarism sometimes occurs due to a misunderstanding regarding the rules of academic integrity, but it is the responsibility of the student to know them. If you are unsure about the standards for citations or for referencing your sources, ask your instructor. Depending on the severity of the case, penalties include a warning, a failing grade, a record on the students transcript, or a suspension.

It is your responsibility to understand the University's policy on academic integrity: http://web.uvic.ca/calendar2018-01/undergrad/info/regulations/academic-integrity.html

How to Succeed in This Course

- Engage with the course material. In this course we will see linear algebra from an abstract, theoretical point of view. A significant part of learning the material will be learning the key definitions and main results that form the foundation for the subject. Understanding these things takes time, and you will be more successful in this course if you commit time to understanding things as soon as they first appear.
- Work out specific examples. While our focus will be on the abstract theory of linear algebra, all of the concepts we will be discussing have concrete manifestations in the case of vectors in \mathbb{R}^n or \mathbb{C}^n . Many of the things we will talk about are things you have already seen in \mathbb{R}^n in Math 211. It is worth your time to take the theoretical developments we cover and see how they work in a more concrete setting. Sometimes we will do this in class, and sometimes on the assignments, but sometimes you will need to do so on your own.
- Start early. The take-home problem sets will be due two weeks from when they are assigned. Some of the questions on these problem sets will be challenging, so you should not expect to be able to finish the problem set the day before it is due. Start the problem sets early so you have plenty of time to think about the problems and get help before the due date.
- Write your work carefully. The majority of the assessments in this course will ask you to write mathematical arguments. In order to obtain full credit for any particular problem your solution will need to be both correct and well-written. In particular, before submitting assignments you should take some time to review what you have written your first attempt at writing a solution should usually not be the



final version you submit for grading. Ask yourself if a person who knew the course material, but wasn't there when you solved the problem, would be able to understand your solution. In this course the intended audience for all of your writing is someone who understands the material in the course so far, but does not believe the result you are claiming to show.

• Get help. Abstract concepts take time to digest, and it is perfectly normal for you to sometimes not understand things the first time you see them. If, after reviewing your notes and consulting the textbook, you find a concept from this course puzzling, please come to my office hours and ask about it. The sooner you clarify things the better prepared you will be for the next topic.

Course Survey

I value your feedback on this course. Towards the end of term you will have the opportunity to complete a confidential course experience survey (CES) regarding your learning experience. The survey is vital to providing feedback to me regarding the course and my teaching, as well as to help the department improve the overall program for students in the future. When it is time for you to complete the survey, you will receive an email inviting you to do so. If you do not receive an email invitation, you can go directly to http://ces.uvic.ca. You will need to use your UVic NetLink ID to access the survey, which can be done on your laptop, tablet, or mobile device. I will remind you nearer the time, but please be thinking about this important activity, especially the following three questions, during the course.

- 1. What strengths did your instructor demonstrate that held you learn in this course?
- 2. Please provide specific suggestions as to how the instructor could have helped you learn more effectively.
- 3. Please provide specific suggestions as to how this course could be improved.

In addition to the formal feedback at the end of the course, I also welcome your comments throughout the term.

