Orbital Anomalies

Florin Diacu

By the end of the 19th century, the famous Canadian-American mathematical astronomer and polymath Simon Newcomb, together with his collaborators at the United States Naval Observatory in Washington D.C., had achieved a precision of one arc second in their predictions of planetary motion [10]. This feat would not have been possible without the progress of celestial mechanics, which had come a long way since its birth in Newton's *Principia* through the work of mathematical giants such as Euler, Lagrange, Laplace, Poisson, Jacobi, and Poincaré. But an issue still bothered Newcomb. Le Verrier—the co-discoverer of Neptune—had already pointed out the quandary several decades earlier [7]. The problem was the motion of Mercury. Its perihelion advance of about 476 arc seconds per century was by almost 43 arc seconds in excess of what celestial mechanics could account for. Two more decades passed before this disagreement between theory and observation was understood within the framework of general relativity. It then appeared that, at least from the descriptive point of view, gravitation had revealed its last secret, and predictions would pose no problems from then on. But a total victory over celestial motions was still far away.

Toward Better Approximations

In the following years, general relativity achieved great success in cosmology and related fields, but it didn't neglect its applications to celestial mechanics either. To explain the perihelion advance of Mercury, relativity had used only a 2-body problem, so the need for a generalization to any number of bodies became a priority. The contributions of Chazy, [5] Levi-Civita [12, 13], Einstein [9], Eddington [8], and many others achieved this goal. Still, their discrete approximations of Einstein's equations are complicated and difficult to handle other than numerically. In spite of this weakness, the current refinements of the post-Newtonian approximations have reached a high level, finding applications in fields such as geodesy, geophysics, and the Global Positioning System (GPS) [6]. Indeed, without these relativistic equations, the errors would render the GPS useless in urban traffic.

Newtonian celestial mechanics also continued to develop because they offered excellent approximations when the bodies moved at low speeds far away from each other. But the classical n-body problem was also needed to cope with the technological development of the 20th century. New challenges, such as the birth of space science and space voyages, forced mathematical astronomers to take into account many other parameters beyond gravitation, such as magnetic effects and solar wind. To know the exact positions of space shuttles, the experts had to introduce relativistic corrections, so the "classical" equations end up looking different from the Newtonian ones. Soon, however, even those highly sophisticated models could not explain some observations. New phenomena now make the experts wonder whether they understand gravity at all.

The Pioneer Anomaly

On March 2, 1972, *Pioneer 10* was launched from Cape Canaveral on an Atlas-Centaur rocket. NASA had invested great hopes in this mission, whose objectives were to study cosmic rays, magnetic fields, solar wind, neutral hydrogen, dust particles; the Jovian aurorae, radio waves and atmosphere, and Jupiter's satellites, especially Io.

Heading in the direction of Aldebaran, the brightest star in the constellation Taurus, *Pioneer 10* was the first spacecraft ever to reach an outer planet. After passing the asteroid belt and surviving intense radiation, in December 1973 it came close to Jupiter on a hyperbolic orbit near the plane of the ecliptic. Ten years later, it passed beyond Pluto. Thus, *Pioneer 10* was the first artificial object to reach the third cosmic speed, and therefore the first space probe to leave the solar system.

Pioneer 11 followed its sister on April 5, 1973. Both missions fulfilled their objectives brilliantly, sending useful data back to Earth long after the expected lifespan of their functionality had ended. But there was a small problem.

The two spacecraft were closer to the Sun than the computations predicted.

The data indicated the presence of a small Doppler-frequency drift, interpreted as being caused by a tiny and (almost) constant acceleration of $(8.74 \pm 1.33) \times 10^{-10}\ m/s^2$ directed toward the Sun [1, 2]. After 30 years of travel, this acceleration delayed the space probes by about 8 hours, or 380,000 km, which is roughly the distance between Earth and Moon. Though this disagreement between theory and data may seem insignificant, our computations are so precise today that this discrepancy should not occur. Also strange is that this orbital anomaly took effect only after the spacecraft passed Saturn. Everybody suspected a single cause for this phenomenon, but nobody knew what it was.

The Model

The determination of the distance to the space probe follows a two-way Doppler-tracking method [11]. According to special relativity, a signal of frequency ν_0 sent from Earth is seen by the probe as having frequency

$$\nu_1 = \frac{(1 - v/c)\nu_0}{\sqrt{1 - v^2/c^2}},$$

where v is the probe's velocity relative to Earth and c the speed of light. After reaching the spacecraft, the signal is sent back to Earth, where its frequency is perceived as

$$\nu_2 = \frac{(1 - v/c)\nu_1}{\sqrt{1 - v^2/c^2}}.$$

Consequently,

$$\frac{\nu_2 - \nu_0}{\nu_0} = -\frac{2v/c}{1 + v/c} \approx -\frac{2v}{c},$$

from which the observed velocity $V_{obs} := v$ of the probe can be computed. Then V_{obs} is compared to the computed velocity, V_{com}, of the spacecraft, leading to the acceleration of $(8.74 \pm 1.33) \times 10^{-10}\ m/s^2$ mentioned earlier.

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The computation of V_{com} takes quite an effort within the framework of a sophisticated model, which combines the Newtonian and relativistic n-body problems with other forces and effects. These ingredients can be briefly described as follows:

- Gravitational forces. The model uses a relativistic 11-body problem that contains the 9 planets (Pluto included) as well as the Sun and the Moon. The Newtonian gravity coming from large asteroids, the Earth tides, and the lunar librations are also taken into account.
- Nongravitational forces. These forces are either external relative to the space probe, such as solar wind, solar radiation, and drag from interplanetary dust, or internal. The latter include control maneuvers and thermal radiation, and the torque that these two forces produce.
- Model of ground stations. Since the observation stations are on Earth, several motions must be taken into account: precession, nutation, sidereal rotation, polar motion, tidal effects, and tectonic plates drift.
- Model of signal propagation. The computations consider a relativistic model for light propagation with order up to v^2/c^2 and the dispersion of the signal because of interplanetary dust and solar wind.
- Computational methods. Four independent algorithms are used to ensure the correctness of the computations. One is an orbit determination programme developed by Jet Propulsion Laboratory, the second a computer code from the Aerospace Corporation, the third an algorithm obtained at Goddard Space Flight Center, and the fourth a programme from the University of Oslo.

In other words, the model used to compute the orbit and velocity of the spacecraft takes into account all the known forces and effects that could have even the slightest influence on the probe's motion. But in spite of this care, the difference between the observed and the computed velocity is not negligible. No wonder that several researchers tried to explain this discrepancy.

Attempted Explanations

A candidate for the slow-down of the probes' motion is the interplanetary and interstellar dust. The former is known to have a density of less than $10^{-24}\ g/cm^3$ and the latter of less than $3 \cdot 10^{-26}\ g/cm^3$. But the calculations show that only a density $3 \cdot 10^5$ larger than that of the interplanetary dust could account for the anomalous acceleration, therefore this attempt to explain the phenomenon failed.

Another attempt used a result from general relativity. The post-Newtonian formalism is based on the assumption that the Sun's centre of mass follows a geodesic. But this is not true in a general relativistic framework. So some authors assumed that the Sun has an acceleration orthogonal to the ecliptic, a result that follows from an exact solution of Einstein's equations, first discovered by Levi-Civita in 1918 [4]. The computations show, however, that this acceleration can explain the Pioneer anomaly only if all radiation of the Sun is emitted in a single direction—a hypothesis that is obviously not satisfied.

The possibility of additional mass in the solar system also raised hopes for an explanation. Apart from dust, additional mass occurs due to larger particles, such as those that form rings around Saturn and Uranus. But again the computations could not account for the anomalous acceleration unless they assumed the additional mass exceeded 100 Earth masses, and that would contradict the observed and calculated orbits of several comets.

A fancier attempted explanation was to link the Pioneer anomaly with cosmic expansion. Since the anomalous acceleration is approximately equal to cH, where c is the speed of light and H the Hubble constant, some researchers thought that the cosmic expansion influences the trajectory of the probes, the magnitude of the gravitational field, signal propagation, or the definition of the Astronomical Unit. But this attempt failed too. The computations showed that the acceleration resulting from cosmic expansion is tiny, namely $VH = (V/c)cH$, which is by a factor of V/c smaller than cH.

Finally, researchers checked to see whether the problem is related to the drift of clocks on Earth. They went as far as to ask if a nonconventional physics might be necessary to explain the Pioneer anomaly. But this approach led to no better understanding either.

The Flyby Anomaly

More recently, another anomalous behaviour was observed—this time close to home. Between 1990 and 2005, several missions were launched in the solar system, each with different objectives. All of them, however, started with flybys around the Earth for the purpose of attaining the right direction and velocity to engage on the desired orbits. For the first of them, *Galileo*, launched in December 1990, the NASA engineers detected a frequency increase, which corresponded to an accelerated motion that found neither a classical nor a relativistic explanation. When, two years later, *Galileo* passed again, it came within 300 km of Earth, so close that atmospheric drag impeded the detection of any anomaly. But two other missions—*NEAR*, launched to study an asteroid, and *Rosetta*, aimed at a comet—experienced the same accelerated motion during their flybys. Initially, thrusting maneuvers for the *Cassini* probe, whose goal was to reach Saturn, prevented the detection of any anomaly. The same thing happened with the *MESSENGER* mission, aimed at Mercury in 2005.

But a team led by John D. Anderson from the Jet Propulsion Laboratory at the California Institute of Technology in Pasedena recently analyzed the data from all six flybys in great detail [3]. These researches found that each spacecraft experienced the same anomalous acceleration. The behaviour was so similar that they could capture its pattern in a single empirical formula, which can now predict future anomalies and help with placing probes on the desired orbits. Unfortunately, nobody knows yet why spacecraft experience this acceleration during flybys. As in the case of the Pioneer anomaly, several attempts have been made to explain this phenomenon.

Failed Attempts

The flyby anomaly corresponds to a velocity increase of a few mm/s, ranging from 1.82 ± 0.05 for *Rosetta* to 13.46 ± 0.13 for *NEAR* [11]. These differences depend, of course, on positions and velocities relative to Earth. Again, these tiny values show how precise our measurements and calculations have become.

A first attempt to explain this mysterious phenomenon was the drag of the atmosphere. The drag acceleration is given by the formula

$$a = -K\rho v^2 A/m,$$

where K is the probe's drag coefficient, which can be safely approximated with 2, ρ is the density of the atmosphere, v the velocity of the spacecraft, A its effective area, and m its mass. For a density $\rho \approx 10^{-14}$ kg/m^3 at a height of 1000 km, a velocity $v = 30$ km/s, an effective area $A = 2$ m^2, and a mass of 1 t, the drag acceleration is of the order 10^{-8} m/s^2, much too small to explain the anomaly and of the wrong sign as well.

Another idea was to check whether ocean or solid Earth tides have any impact on the change in velocity of the spacecraft. The acceleration caused by tides turns out to be at most 10^{-5} m/s^2, again too small to provide an explanation. The solid Earth tides are much smaller than the ocean tides, so they cannot account for the flyby anomaly either.

The Earth's albedo accounts for an effect of 10^{-4} m/s^2, the charging of the probe with electricity an effect of at most 10^{-8} m/s^2, and the magnetic moment an effect of only $4 \cdot 10^{-15}$ m/s^2—all three of them much too small as compared with the unexplained acceleration. The effect from the solar wind is also negligible, exercising an influence of less than $3 \cdot 10^{-9}$ m/s^2. The Earth's oblateness, the Moon and its oblateness, the Sun, and the gravitational attraction of the other planets were also taken into account, but all turned out to be of at least one order of magnitude smaller than the anomalous acceleration.

Other researchers looked at alternative models. One considered a potential energy that contained the time explicitly. Another one used a non-Newtonian classical model for gravity. A third modified slightly the relativistic model, and a fourth succeeded in matching the data from the probes but failed to explain the current relative stability of the planetary orbits, so the model proved unsuitable for the solar system.

The experts hope that future missions will provide more data and perhaps hint at a research direction that could explain both the Pioneer and the flyby anomalies. Whether gravity alone is responsible for these phenomena is not clear. The possibility remains at this time that only some new physics will provide an explanation.

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Why Fly By?

We have all heard that a spaceship can use a close encounter with a planet to pick up energy enabling it to visit distant parts of the Solar System. Most accounts don't explain the physics of this "boost".

It makes one pause for thought, doesn't it? We know that the spaceship passing Earth in empty space follows an orbit that is a conic section with Earth at one focus. That orbit is symmetrical: at any given distance as it is leaving the vicinity of Earth, the ship's speed is just the same as when it was at that distance on its approach. No boost there!

Let me try to sort this out. The effect is not deep, and no relativity is involved, still less the mysterious anomalies now perplexing experts.

Let the mass of the Earth be m and that of the Sun be M. We can choose units, so let the mass of our spaceship be 1. We must assume $1 << m << M$. Assume all three bodies are moving in a plane.

Briefly during the flyby, we will be so close to Earth that the Sun's effect on our course will be negligible: our path will be a hyperbola with Earth at one focus. During most of our flight, the Earth's effect on our course will be small: our path will be approximately an ellipse with the Sun at one focus (not a hyperbola, for we are short of energy and surely are not bound for Aldebaran). These simpler situations are best parametrized in terms of the classical elements of the conic sections involved.

The elliptical orbit around the Sun is characterized conveniently by its semi-major axis a and its semi-minor axis b. Better yet, let's use a and the eccentricity

$$e = \sqrt{a^2 - b^2}/a \in [0, 1).$$

The reason these are convenient parameters is that we can easily express the range of values for the speed. The ship's potential energy is, as usual, $V = -GM/r$, and its kinetic energy $K = v^2/2$; here G is the universal constant of gravitation, r the distance from the Sun, and v the speed. Though r and v vary, the total energy $E = K + V$ remains constant throughout the orbit. As we are reminded elsewhere in this issue in "Teaching the Kepler Laws for Freshman" by Gert Heckman and Maris Van Haandel, this constant value is $E = -GM/2a$. Now at perihelion, where r takes its minimum value $a(1 - e)$, K takes its maximum $E + GM/(a(1 - e))$, from which we compute

$$v^2_{\max} = \frac{GM}{a}\frac{1+e}{1-e}.$$

Similarly, the minimum of K and hence of v occurs at aphelion, and we compute

$$v^2_{\min} = \frac{GM}{a}\frac{1-e}{1+e}.$$

(Of course, to describe the orbit completely needs more parameters: an angle to tell in which direction the ship would make its closest approach to the Sun (the "apsidal line"), and something to tell *when* it would do what. But a and e suffice for specifying the shape of the orbit, and tell neatly what speeds are possible.)

Thus before it passes by Earth our ship is on one ellipse, with parameters a, e, and after the encounter it is on another, with parameters a', e'. What happens in between?

When it is near the Earth, our path is a Kepler orbit around the Earth, but as the ship has enough energy to "escape" Earth's field, the orbit is a hyperbola with the Earth at one focus. The potential energy relative to Earth is $V = -Gm/r$ with the same G as previously described but a different mass m, and now r denotes the distance to the Earth. We approach, essentially, along one asymptote of the hyperbola and leave along the other, with energy unchanged. The flyby has switched us from one elliptical orbit around the Sun to another one, with the same speed (relative to Earth) but a different direction.

By the way, in putting it so simply I have ignored the Earth's revolution about the Sun. This actually loses nothing: if we converted to a coordinate frame rotating with a period of a year, only minor changes in the parameters would result, the picture would stay the same. The only fraud in the preceding paragraph is that it said nothing about those transitional interludes when the ship is too close to Earth to ignore its gravitational field but not close enough to ignore the Sun's. There, for a while, the problem is intractably the 3-body problem, and I am silent.

This short-cut of skipping from one Newtonian orbit to another is known by the natural name of *patched conics* to specialists in designing space missions—so I am told by one of them, Jeremy Kasdin of Princeton. He also recommends two textbooks: *Fundamentals of Astrodynamics* by R.R. Bate, D.D. Mueller, and J.E. White; and *Modern Spacecraft Dynamics and Control* by M.H. Kaplan.

Consider our flyby, then, as giving the spaceship only a change in heading, with speed (of ship relative to Earth) staying fixed. That speed is given us, but we can choose the amount of change in heading almost freely, by really small changes in the line along which our orbit approaches Earth. If $\mathbf{v}$ and $\mathbf{v}'$ are the velocities (relative to the Sun) before and after, and $\mathbf{u}$ the velocity of Earth relative to the Sun, then the restriction is that $|\mathbf{v}' - \mathbf{u}| = |\mathbf{v} - \mathbf{u}|$, which leaves room for v' to exceed v by at most $2u$. Still—the potential energy relative to the Sun has remained the same, whereas we can arrange for the speed to have increased by something on the order of $|\mathbf{u}|$, increasing the kinetic energy; so the total energy will have increased. Also, we can arrange for the new eccentricity e' to be very close to 1. Now the farthest our ship will be able to get from the Sun on the new orbit is $a'(1 + e')$. We have seen that a' may be boosted and that $1 + e'$ can be brought up close to 2. There is indeed something to be gained by this mysterious maneuver the flyby.

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