Editorial

Mathematical Research: Invention and Discovery

by Florin Diacu, Guest Editor
University of Victoria

This issue of *Pi in the Sky* celebrates the work of Leonhard Euler—the most prolific mathematician ever. At his death, in 1783, he left so many unpublished manuscripts behind that it took more than half a century to print them. The ongoing publication of his collected works, which started in 1910, will fill an estimated 80 large quarto volumes. His research touched most branches of pure mathematics, from number theory to calculus, and dealt with applications in various fields, some as far apart from each other as music theory and celestial mechanics.

But what is mathematical research, and how is it done? Many people are surprised to learn that such an enterprise exists. One of them asked me once: "Are you reinventing the numbers, or what?" In his view, our only job was to teach students, and was appalled that we lectured only a handful of hours per week. He didn't know how fully committed we are to finding new theorems, often sparing no nights or weekends in search of results.

In a way, our work resembles what students do. Like them, we use our knowledge and skills to answer mathematical questions. We follow logical reasoning, employ methods, and apply computational techniques to achieve our goals. But here the analogy ends. Unlike students, we cannot check the answer key to our unsolved problems. Often we have no clue how to approach a question or get stuck in every direction we take. Sometimes we have to invent novel methods because the old ones don't help.

Most problems we attempt are hard. We may need weeks or months to make progress with them. If the results are significant enough, we publish them, thus giving others a chance to take our work further. Some

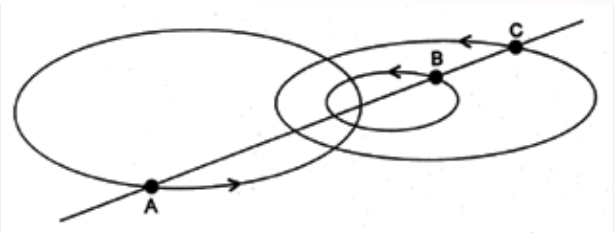
Leonhard Euler in 1753--as portrayed by the Swiss painter Emanuel Handmann.

questions, however, are so difficult that only collective efforts of years or decades lead to answers. And there are problems that have eluded our understanding for centuries.

One of the most celebrated unsolved mathematical questions is the 3-body problem of celestial mechanics, which Isaac Newton posed in 1686. Given three point masses that move in space under the influence of gravity—the problem states—determine their trajectories for any choice of initial positions and velocities. Many famous mathematicians contributed to this question, Euler among them (see the pink box), but after more than three centuries and thousands of published papers, this problem continues to elude our understanding. Still, what we achieved so far allows us to compute the orbits of planets, asteroids, and comets, and to design missions in space. From the theoretical point of view, this problem led to the creation of new branches of mathematics, such as algebraic topology and dynamical systems, and to the discovery of chaos, an intriguing mathematical property encountered in many time-evolving phenomena, the weather and the stock market among them.

Like the 3-body problem, other questions inspire researchers and lead to the development of mathematics, a world open to everyone who has the desire and strength to enter it. No doubt, the young readers of this magazine are eager to explore realms of thought nobody has conquered before. I trust that, someday, many of them will.

The Eulerian solution of the 3-body problem



In 1767, Euler proved that if one assigns suitable initial positions and velocities to 3 point-masses, A, B, C, among which gravitation acts, they move forever along ellipses without leaving a rotating straight line.