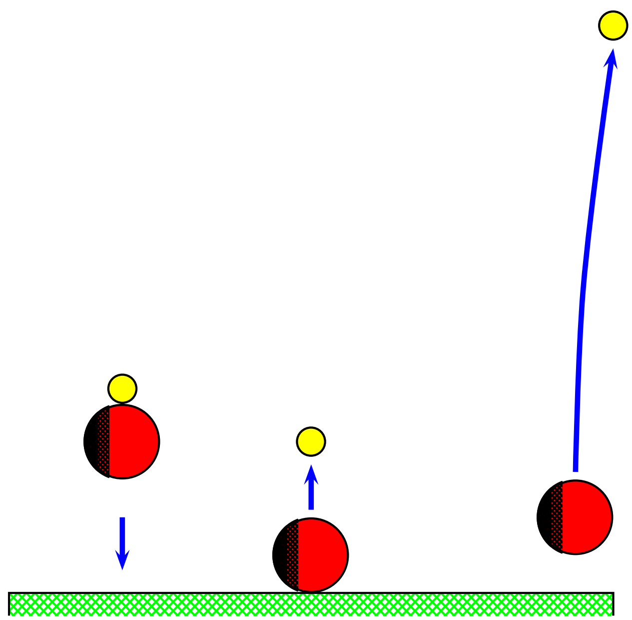
Math Discovery

The Slingshot Effect of Celestial Bodies

by Florin Diacu†

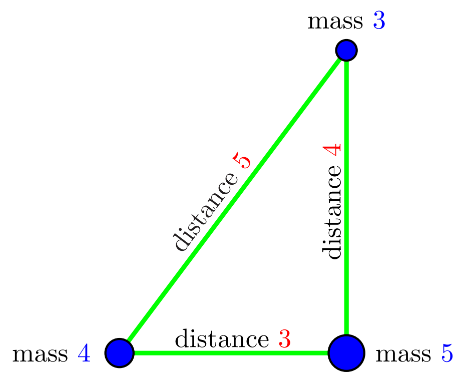
Try the following outdoor experiment: place a tennis ball above a basketball and let them fall from the level of your shoulders. The outcome is unexpected; the bounce of the basketball will send the tennis ball at high speed well above your head. This surprising energy-transfer phenomenon has recently been under careful mathematical investigation. Its full understanding will help us find new fuel-free accelerating techniques for space shuttles and ways of travelling astronomical distances faster.



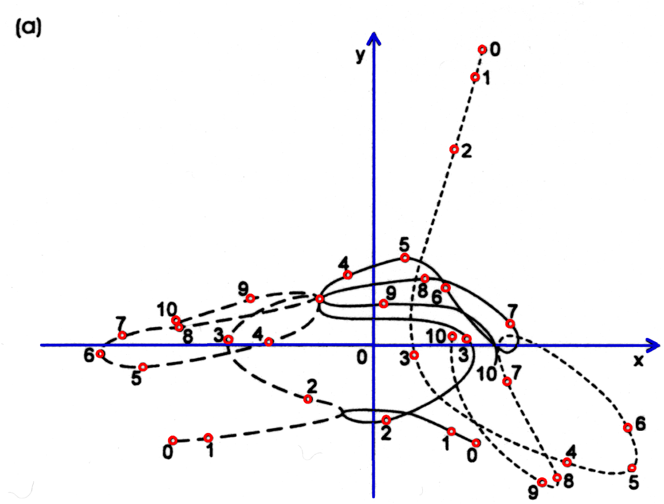
Everything started in 1687, when Isaac Newton published his masterpiece *Principia*, in which he founded several branches of science, including calculus (studied today by freshmen students), the theory of differential equations (which is part of the sophomore sicence curriculum), and celestial mechanics. The first two apply to many fields of human activity ranging from physics and economics to psychology and art. Based on these mathematical theories, celestial mechanics aims to understand the gravitational motion of stars, planets, asteroids, and comets, and to compute the orbits of spacecrafts.

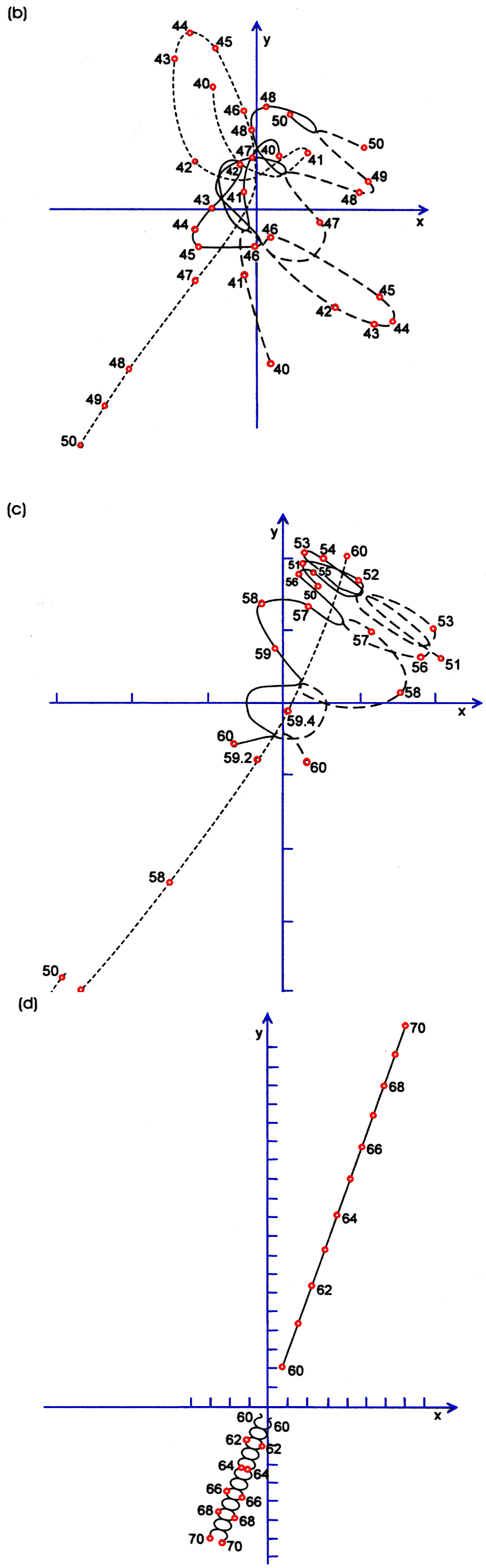
Since Newton, many famous mathematicians like Bernoulli, Euler, Lagrange, Laplace, Gauss, Jacobi, Poincaré, Birkhoff, and others, tried to predict the trajectories of celestial bodies. But the differential equations describing the orbits are so complicated that any hope of obtaining a complete solution was abandoned long ago. Still, interesting results come up from time to time.

One of them shows that the close encounter of 3 celestial bodies leads to a slingshot effect as in the earth-tennis-basketball experiment. This property was discovered in 1966 through computer simulations done by Victor Szebehely and Myles Standish at Yale University and by Eduard Stiefel at ETH-Zurich. From the vertices of a triangle having 3-, 4-, and 5-length-unit sides, they released bodies of 3-, 4-, and 5-mass units. Gravitation first led to an erratic behavior, but then made 2 bodies orbit around each other and the third move away at high speed (see the picture below).



These computer results can be followed in the graphs represented below, in which the orbits of the 3-, 4-, and 5-mass-unit bodies are drawn as dotted, dashed, and continuous lines, respectively. The numbers along each line denote time units; they allow us to locate each particle. The graphs (a) and (b) show a complicated motion without any pattern, graph (c) indicates that the heavier particles tend to move around each other, and graph (d) makes clear how the lighter particle is expelled with high velocity away from the other two after a close triple encounter (which takes place between the time units 59 and 60).





In 1973 Jörg Waldvogel from ETH-Zurich initiated the computer investigations of the general motion of 3 bodies. Again, after coming close to each other, the bodies first moved unpredictably and then followed the slingshot pattern. Ignorant of these results, Richard McGehee of the University of Minnesota in Minneapolis rigorously proved in 1974 that, if on a line, the bodies must encounter a slingshot effect near triple collision.

These results led to the theoretical discovery of motions that become unbounded in finite time. In other words, celestial bodies could move so fast that they reach infinity in a couple of seconds. How can this happen? Imagine that a body is accelerated through consecutive slingshot effects such that it travels the first mile in a second, the second mile in half a second, the third mile in a quarter of a second, etc. This means that after $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$ seconds the body travels infinitely many miles. But mathematicians know that the above infinite sum has the value 2. So as inconceivable as it may be, a body could reach infinity in only 2 seconds.

The difficulty was to prove that a proper sequence of slingshot effects can lead to the above scenario. The success came in 1992 when Zhihong Xia, a young Ph.D. student at Northwestern University, published his thesis. In his work he showed that, if properly positioned, 5 bodies can move under the influence of gravitation such that 4 of them escape to infinity in finite time, while the fifth oscillates back and forth among the others. At a recent meeting in Vancouver, Xia received the Blumenthal Award, made every 4 years in recognition of distinguished achievements in mathematics.

In the mean time space scientists tried to use the slingshot effect for practical purposes. Through related work done in 1991, Edward Belbruno, a consultant with Jet Propulsion Laboratory in Passadena, California, managed to find a Japanese sattelite lost several months earlier and to propose a mathematical solution for rescuing it. The Japanese used this solution to succeeded in their mission, an event that made headlines in 1994.

The slingshot effect is an intriguing phenomenon, still under the scrutiny of mathematicians and space scientists. It is amazing how from a simple observation of a ballgame experiment, intense and dedicated mathematical research can lead to such spectacular achievements.

About the Author: **Florin Diacu** is a mathematics professor at the University of Victoria and the UVic-Site Director of the Pacific Institute for the Mathematical Sciences. His award-winning book *Celestial Encounters*, co-authored with Philip Holmes of Princeton University, is a runaway bestseller. You can find more information at:

www.math.uvic.ca/faculty/diacu/index.html