2023 Coast Combinatorics Conference

November 4, 2023
University of Victoria
Clearihue Building, room A127

9:30 **COFFEE** can be had at The Cove

10:00 *The Rainbow Connection*, Shannon Ogden, University of Victoria

Given a graph H, an edge-coloured graph G is *H-rainbow saturated* if it does not contain a rainbow copy of H, but the addition of any non-edge in any colour creates a rainbow copy of H. The *rainbow saturation number*, denoted by $\mathrm{rsat}(n, H)$, is the minimum number of edges among all H-rainbow saturated edge-coloured graphs on n vertices. We prove that, for any non-empty graph H, the rainbow saturation number is linear in n. This confirms a recent conjecture of Girão, Lewis, and Popielarz.

Based on work with Natalie Behague, Tom Johnston, Shoham Letzter, and Natasha Morrison.

10:30 *Cyclic Orderings of Paving Matroids*, Sean McGuinness, Thompson Rivers University

A matroid M of rank r is *cyclically orderable* if there is a cyclic permutation of the elements of M such that any r consecutive elements form a basis in M. An old conjecture of Kajitani, Miyano, and Ueno states that a matroid M is cyclically orderable if and only if for all $\emptyset \neq X \subseteq E(M)$, $\frac{|X|}{r(X)} \leq \frac{|E(M)|}{r(M)}$. In this talk, we will show that this conjecture is true for all paving matroids.

11:00 *Turan Colourings in Ramsey Multiplicity*, Jae-Baek Lee, University of Victoria

The Ramsey multiplicity constant of a graph H is the limit as n tends to infinity of the minimum density of monochromatic labelled copies of H in a colouring of the edges of K_n with two colours. Fox and Wigderson recently identified a large family of graphs whose Ramsey multiplicity constants are attained by sequences of "Turan colourings". The graphs in their family come from taking a connected non-3-colourable graph with a critical edge and adding many pendant edges. We prove that the monochromatic copies of the disjoint union of K_4 and any non-empty forest is minimized by Turan colourings. Our proof involves an application of flag algebra method and uses the famous Clique Density Theorem of Reiher.

This is joint work with Joseph Hyde and Jonathan Noel.

11:30 *Operator Constraint Satisfaction Problem and Satisfiability Gap*, Andrei Bulatov, Simon Fraser University

The Constraint Satisfaction Problem (CSP) asks to find a satisfying assignment to a collection of variables subject to specified constraints. Graph Coloring and Graph Homomorphism problems are standard examples of CSPs. In the CSP and other homomorphism problems we are looking for an assignment of discrete values such as colors of vertices of a graph. However, one may follow the lead of quantum mechanics and convert bits to qubits, that is, look for satisfying assignments by matrices or linear operators. More precisely, constraints in a CSP are interpolated by polynomials (real or complex) that allows for using linear operators as values of variables. Sometimes a CSP that is not satisfiable 'classically' may have a satisfying operator assignment. In this talk we study what kind of CSPs demonstrate such a satisfiability gap.

12:00 **LUNCH BREAK** on your own

1:30 *The Damage Number of the Product of Graphs*, Melissa Huggan, Vancouver Island University

In adversarial situations on networks, we often concern ourselves with minimizing resources required for neutralizing a threat. Here we consider a different parameter which addresses the situation where the aggressor is damaging each unique location they visit. Framed within the context of the game of Cops and Robbers on graphs, the robber tries to maximize the number of unique vertices they visit in order to maximize the damage to the graph, while the cops aim to minimize the damage by limiting the robber territory. This model was first introduced in 2019 by Cox and Sanaei. We build on their results. We provide a general upper bound for the damage number of the Cartesian product of graphs and consider the damage number of the product of two trees or cycles. We also consider graphs with small damage number along with their products.

This is joint work with Margaret-Ellen Messinger and Amanda Porter.

2:00 *Balanced Weighting Matrices*, Thomas Pender, Simon Fraser University

A weighing matrix is a square $(-1, 0, +1)$-matrix with pairwise orthogonal rows such that each row has a constant number of non-zero entries. If upon taking the absolute values of all the entries of a weighing matrix, one obtains an incidence matrix for a symmetric balanced incomplete block design, we say that the weighing matrix is balanced. In this talk, we construct a new infinite family of balanced weighing matrices.

This is joint work with Dr. Hadi Kharaghani (University of Lethbridge) and Dr. Sho Suda (National Defense Academy of Japan).

2:30 *Proper Rainbow Saturation*, Andrew Lane, University of Victoria

Given a graph H, say that a graph G is properly rainbow H-saturated if there exists a rainbow H-free proper edge-colouring of G and, for any non-edge e of G, every proper edge-colouring of G+e contains a rainbow copy of H. The proper rainbow saturation number is the minimum number of edges in a properly rainbow H-saturated graph on n vertices. This is a natural variant of the graph saturation problem based on the rainbow extremal number. In this talk, we prove bounds on the proper rainbow saturation number for specific classes of graphs, and we prove general bounds on the proper rainbow saturation number using related saturation numbers.

Joint work with Natasha Morrison.

3:00 **COFFEE** can be had at BiblioCafe

3:30 *Promise Constraint Satisfaction Problem*, Arash Beikmahammadi, Simon Fraser University

The Approximate Graph Coloring Problem (AGCP) is a decision problem where for a given graph $\mathbf{G}$ that is either k-colorable or not even c-colorable for $c \geq k$, one must decide between these two cases. Despite its apparent simplicity, AGCP remains a challenging problem with a conjectured NP-hardness for all constants $3 \leq k \leq c$, yet only limited progress has been made in resolving its computational complexity. A natural extension of AGCP, known as the Promise Constraint Satisfaction Problem (PCSP) has been recently introduced. For two fixed relational structures $\mathbf{A}$ and $\mathbf{B}$, $PCSP(\mathbf{A}, \mathbf{B})$ is a decision problem where a relational structure $\mathbf{I}$, similar to both $\mathbf{A}$ and $\mathbf{B}$, is given as input, and one must decide whether it has a homomorphism to $\mathbf{A}$ or does not have any homomorphism to $\mathbf{B}$. Building upon recent advancements in PCSP, AGCP has been proved to be NP-hard for $c = 2k - 1$, $k \geq 3$ and $c = \binom{k}{\lfloor \frac{k}{2} \rfloor}$, $k \geq 5$. Recetly, some relations between promise CSP and algebraic topology have been found which led to the NP-hardness of $PCSP(\mathbf{C}_{2c+1}, \mathbf{K}_3)$ for $c \geq 1$.

4:00 *Tight Upper Bounds for the Anionic Clar Number of Fullerenes*, Aaron Slobodin, University of Victoria

A fullerene is an all-carbon molecule with a polyhedral structure where each atom is bonded to three other atoms and each face is either a pentagon or a hexagon. Fullerenes correspond to 3-regular planar graphs whose faces have size five or six.

The *p-anionic Clar number* $C_p(G)$ of a fullerene G is equal to $p + h$, where h is maximized over all choices of $p + h$ independent faces (exactly p pentagons and h hexagons) whose deletion leaves a graph with a perfect matching. This definition is motivated by the chemical observation that pentagonal rings can accommodate an extra electron, so that the pentagons of a fullerene that has negative charge $-p$ compete with the hexagons to host 'Clar sextets' of six electrons, and pentagons will preferentially acquire the p excess electrons of the anion.

The *Clar number* is equivalent to the 0-anionic Clar number and has been studied extensively. Little work has been done for other values of p, as this parameter has only recently been defined.

This talk presents tight upper bounds for the p-anionic Clar number when p is not zero. We also explore the feasibility of our graph theoretic solutions for the anionic Clar model when compared against several chemical models for charged fullerenes.

This is joint work with Patrick Fowler, Gary MacGillivray, and Wendy Myrvold.

4:30 *Eternal Distance-d Domination in Trees*, Alex Clow, Simon Fraser University

This talk considers joint work with Christopher van Bommel (University of Guelph) on the eternal distance-k domination problem recently proposed by Cox, Meger, and Messinger. In particular we will present novel upper and lower bounds on the eternal distance-k domination number $\gamma^{\infty}_{all,k}$, generalising work by Chambers, Kinnersley, and Prince. In order to show both bounds are tight we construct a family of trees meeting the given upper bound, and another which meets the given lower bound. Next we provides a fast algorithm for computing $\gamma^{\infty}_{all,2}$ in trees, generalising work by Klostermyer and MacGillivray. Additionally we explore why our method is insufficient for computing $\gamma^{\infty}_{all,k}$ in trees for $k > 2$ and conclude with a list of open problems.

5:00 **END**