

# My Favourite Mathematician Leonhard Euler

Kelly Choo

September 20, 2012

## Abstract

This is an article about Leonhard Euler. After a brief biography of this great Swiss mathematician, three of his well known results will be discussed. Lastly, we list some conventional math notation first used by Euler.

## 1 A Short Biography

Leonhard Euler was born in Basel, Switzerland, in 1707. He is known to be the most prolific mathematician having published 530<sup>1</sup> books and papers during his lifetime. His achievements are made that much more remarkable by the fact that he was totally blind during the last 17 years of his life. He died in 1783.

## 2 Well Known Results

This section talks about three of my favourite results attributed to Euler.

### 2.1 A Beautiful Formula

One of the most beautiful formulas known to mathematicians is

$$e^{ix} = \cos x + i \sin x. \tag{2.1}$$

---

<sup>1</sup>From [1]

When  $x = \pi$ , equation 2.1 becomes

$$e^{i\pi} + 1 = 0,$$

a relation connecting the five most important numbers in mathematics.

## 2.2 The Seven Bridges of Königsberg

In the Prussian town of Königsberg, there are seven bridges over the Pregel river which connect the town's four districts (two riverbanks and two islands). Citizens of the town often wondered whether it was possible to walk through the town and cross over each of the seven bridges once and only once.

Euler solved the problem by using a graph, whereby the four land masses are represented as "dots" and the seven bridges are represented as *lines*. A line would connect two dots if there is a bridge connecting two land masses. He showed that the circuit through Königsberg as described above is impossible to do.

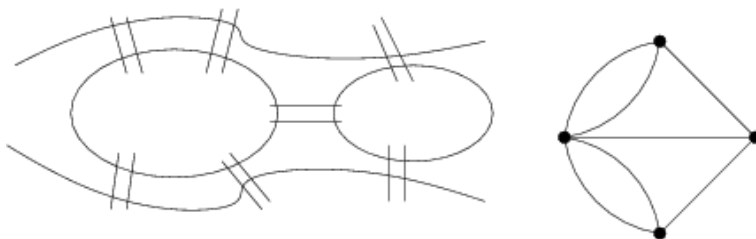


Figure 1: The Seven Bridges of Königsberg (from [2]).

**Theorem 2.1** *A graph with at least two vertices has an Euler path if it is connected and it has at most two vertices of odd degree.*

## 2.3 Faces, Edges and Vertices

Euler came up with the relation

$$v - e + f = 2$$

connecting the number of vertices  $v$ , edges  $e$  and faces  $f$  of any simple closed polyhedron. The following table shows the values of  $f$ ,  $e$ , and  $v$  of the five platonic solids.

<b>Platonic Solid</b>	$f$	$e$	$v$
<i>Tetrahedron</i>	4	6	4
Cube	6	12	8
Octohedron	8	12	6
Dodecahedron	12	30	20
Icosahedron	20	30	12

### 3 Notation

In this section we list some of the notation conventionalized by Euler.

1.  $f(x)$  for function notation,
2.  $e$  for the base of natural logarithms,
3.  $i$  for the imaginary number  $\sqrt{-1}$ ,
4.  $\Sigma$  for the summation sign.

### References

- [1] H. Eves: *An Introduction to the History of Mathematics*, Saunders College Publishing, Fort Worth, 1990.
- [2] D. West: *An Introduction to Graph Theory*, Prentice-Hall, Upper Saddle River, New Jersey, 1996.