Triangle Decompositions

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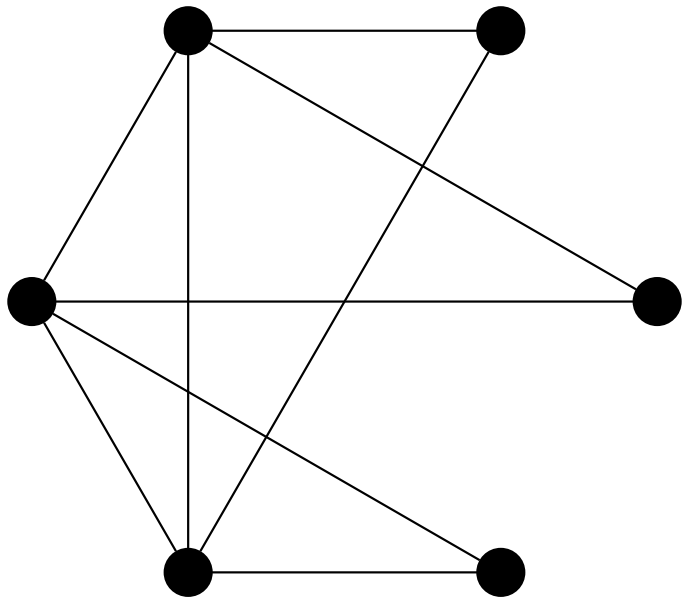
Graphs

A *graph* has a set of vertices (usually drawn as dots) with some edges (lines), each of which connects two vertices.



Triangle decompositions

Question: When can the edges be grouped into triangles? (Triangles can cross each other or touch at corners, but can't overlap on a whole edges!)

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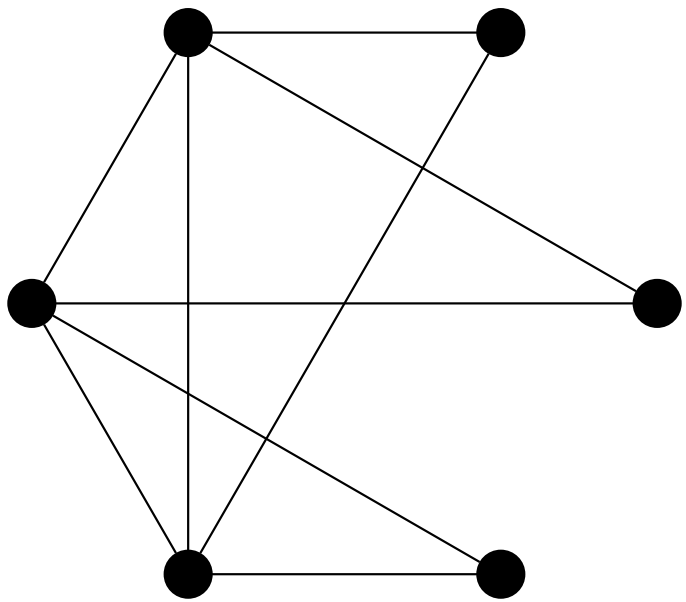
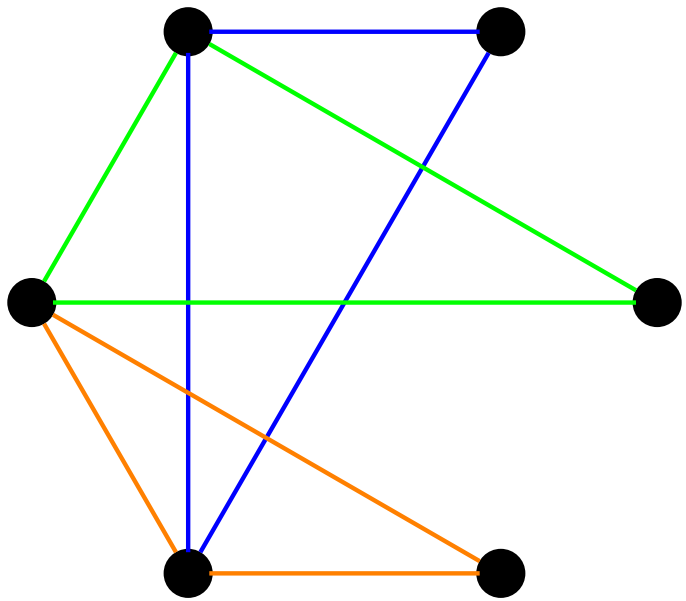


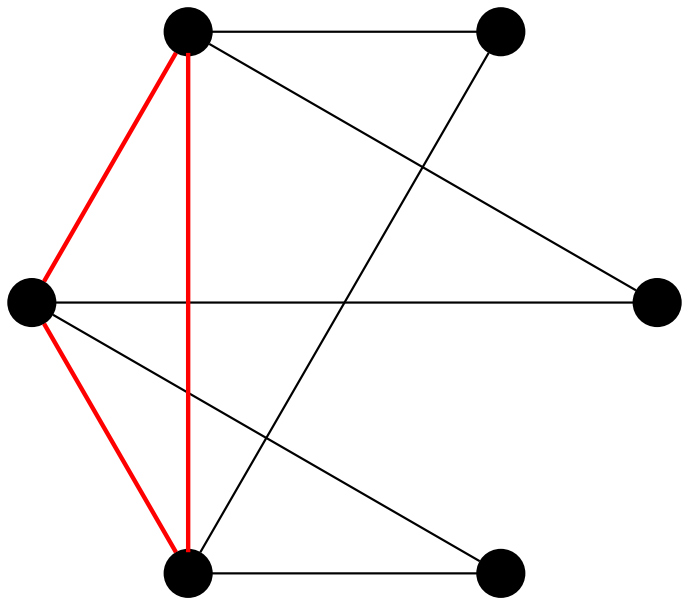
Triangle decompositions

If you succeed, you have found a *triangle decomposition* of the graph.



Oops

Sometimes, you might pick a triangle and find that you need to back up and start over.



Arithmetic conditions

For a graph to have a triangle decomposition, its number of edges must be a **multiple of three**.

Also, the number of edges touching each vertex (called its *degree*) must be **even**.

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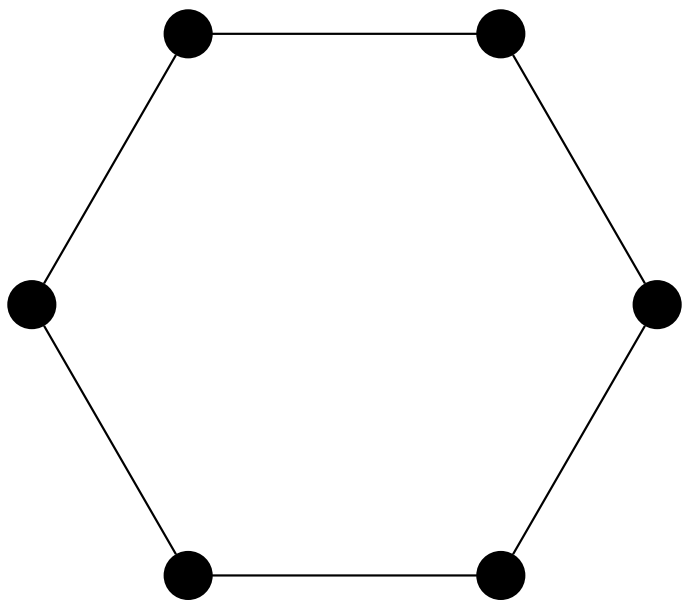
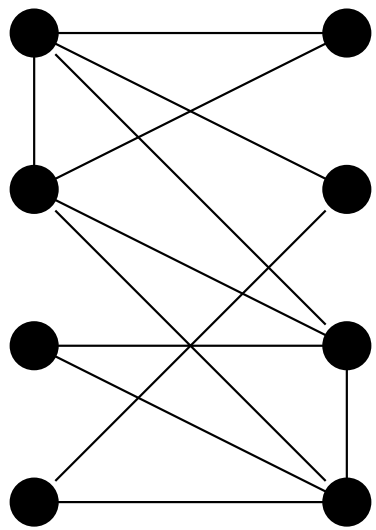
But these conditions are not enough:



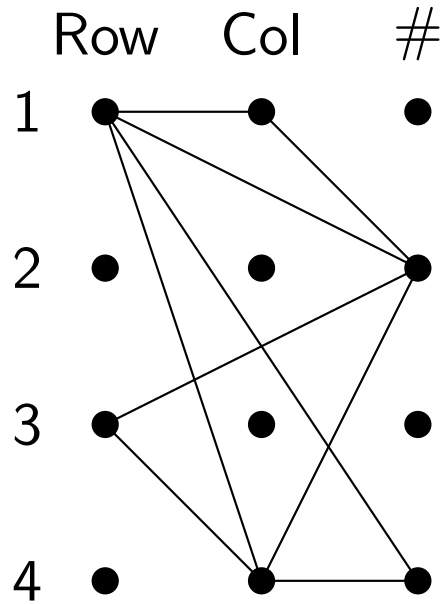
A geometric condition

Another way in which a graph might have no triangle decomposition is that the vertices can be divided into two sets a way that there are too many crossing edges.



Sudoku connection

	1	3	
1	2	4	3
3	4	1	
4	3	2	1



A guarantee for dense graphs

Theorem (Delcourt and Postle, 2019)

Suppose G is a large graph with

- *number of edges a multiple of three;*
- *an even number of edges touching each vertex; and*
- *every vertex joined to at least 83% of the others.*

Then G has a triangle decomposition.

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Then G has a triangle decomposition.

It is conjectured that 83% can be lowered to 75%.

Try out some worksheets, activities and games.

`https://www.math.uvic.ca/~dukes/tridec.html`

Have fun and thank you for watching!

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