UVic Mathematics Competition  
September 26, 2017

- No calculators, books or notes are allowed.
- Write solutions in the booklets provided. Clearly separate rough work from solutions.
- All the necessary work to justify an answer and all the necessary steps of a proof must be shown clearly to obtain full credit.
- Partial credit will be given only for substantial progress toward a solution.
- Questions are of equal value.

Duration: 2 hours

**Question 1.** Given are $n$ fair coins, where the $j$th coin has value ‘1’ printed on one side and ‘$2^j - 1$’ on the other, $j = 1, \ldots, n$. All $n$ coins are flipped, and the product of the face-up values is recorded as a score. Determine the expected score, that is, the average score over all possible flips.

**Question 2.** Consider a row of 2017 switches, whose initial states are: off, on, off, on, \ldots, off. A legal move consists of changing the state of each of two consecutive switches. Using only a sequence of legal moves, is it possible to arrive at the situation in which the middle switch is on and all others are off?

**Question 3.** A circular arc divides the interior of a circle with radius 1 into two regions of equal areas. Prove that the length of the arc is greater than 2.

**Question 4.** Let $p(x)$ be a polynomial with real coefficients, and suppose $p(n) \geq 0$ for each integer $n$. Show that $p(x)$ is a sum of polynomials of the form $a(x)a(x + 1)$. 