UNIVERSITY OF VICTORIA MATHEMATICS COMPETITION

1 October 2012

- Prove all your assertions.
- Partial credit is given only for substantial progress toward a solution.

1 (a) If S is a set of real numbers containing at least 8 elements, prove that there exist three distinct members a, b, c of S such that none of $a+b$, $b+c$, $c+a$ is in S.

(b) Find a set S of real numbers with 7 elements that does not satisfy the property in part (a)

2. For every positive integer n, prove that there exists an n-digit positive integer all of whose digits are odd and which is divisible by 5^n.

3. Determine all strictly increasing functions f from the reals onto the reals such that $f(x) + f^{-1}(x) = 2x$ for every real number x, where f^{-1} is the function-composition inverse of f, i.e. $f^{-1}(f(x)) = f(f^{-1}(x)) = x$

4. Find the minimum side length of a square containing 5 non-overlapping unit squares.