Notation within complete sentences.
Mathematical notation always fits within a sentence. For each of the sentences below, identify all of the verbs:

1. $\lim_{x\to 0} \frac{\sin(x)}{x} = 1$.
2. $42 > -3$.
3. Let $f(x) = g(x) - 2$.
4. If a, b, and c are real numbers then $a(b+c) = ab + ac$.

Each of the following samples contains both grammatical errors and composition problems. Identify as many issues as you can; how would you correct them?

Sample 1
If the degree of a polynomial is > 2 then it has at least one real root and can be factored over the real numbers. This means that the only irreducible polynomials in $\mathbb{R}[x]$ have degree $0, 1$, or 2.

Sample 2
Suppose $a < b \in \mathbb{R}$ and $f(x)$ is differentiable on (a, b) and continuous on $[a, b]$. Rolle's Theorem says that if $f(a) = f(b)$ then $\exists c \in (a, b)$ so that $f'(c) = 0$. Notice that Rolle's Theorem is an existance statement only; it does not tell us what c equals. C is not always easy to find!

Sample 3
Suppose we have a limit that is the form "$0 \cdot \infty$", which is indeterminate.

$$\lim_{x\to\infty} x \ln\left(\frac{1}{x}\right)$$

Most such limits can be rewritten to take the form $\frac{0}{0}$ or "$\frac{\infty}{\infty}$" instead, which is then suitable for l'Hospital's Rule.

$$\lim_{x\to\infty} \frac{\ln(1/x)}{1/x} \qquad \lim_{x\to\infty} \frac{x}{\frac{1}{ln(1/x)}}$$

Notice that we may assume $x > 0$ because $x \to \infty$, so x can be assumed to be large, so all of these fractions are defined. Although you could apply l'Hospital's Rule to either of these limits, usually one of them will be easier to work with than the other. To make our choice, consider whether it is easier to $\frac{d}{dx} \ln(1/x)$ and $1/x$, or x and $\frac{1}{\ln(1/x)}$.

Sample 4
Recall the quotient rule from earlier, which we mentioned at the time could be hard to memorize. Now that we have the chain rule as well, however, we can treat any quotient as a product instead. For example, $\frac{\ln(x)}{\cos(x)} = \ln(x)\,(\cos(x))^{-1}$, $\frac{1}{x}\,(\cos(x))^{-1} + \ln(x)\left(-(\cos(x))^{-2}\,(-\sin(x))\right) = \frac{1/x}{\cos(x)} + \frac{\ln(x)\sin(x)}{\cos^2(x)} = \frac{\frac{\cos(x)}{x}}{\cos^2(x)} + \frac{\ln(x)\sin(x)}{\cos^2(x)} = \frac{\cos(x)\cdot\left(\frac{1}{x}\right) + \ln(x)\sin(x)}{\cos^2(x)}$. When using the product rule and chain rule, we would usually stop at the point $\frac{1/x}{\cos(x)} + \frac{\ln(x)\sin(x)}{\cos^2(x)}$; the remainder of the calculation is there to demonstrate that this method produces the same answer the quotient rule approach would.

Style Francis Su's Guide for Good Mathematical Writing is a fantastic introduction:
The Grammar According to West is a detailed (and in some places a bit inflammatory) resource.

Conventions.

Some conventions aren't required grammatically, but exist as a kindness to your readers or as a sign of formality.

Fundamentally, our goal is to take care of the reader. In the mathematics culture, that means:

- Be clear and succinct. (Hemmingway, not Dickens)
- Use notation when appropriate, in a way that is easy on the eyes.
- Avoid tension.
- Signpost clearly; support your reader when they want to skim.

Here are some specific conventions; for each one I have provided a writing sample that violates it. How would you improve each example?

- Avoid starting a sentence with notation - it can be hard to see where the previous sentence ended, and notation is not compatible with capitalization conventions.

  "$d(v)$ is the degree of the vertex v."

- Always have at least one English word between separate pieces of notation, so your reader can easily see the separation.

  "For any $x \in \mathbb{R}$, $x^2 \geq 0$."

- Explicitly state "if" and "then" where you can (this is not the usual English composition advice).

  "If f is differentiable at a point, f is continuous at that point."

- Be careful with the words "as" and "for". They have specific mathematical uses, which means their English uses can be confusing in mathematical writing.

  "We will discard $f(x_i)$, for x_i is not positive."

  "As f grows, its derivative must be positive."

- Symbols such as $=, \leq, >$ must only be used to compare like with like.

  "We will first consider polynomials of degree ≥ 3."

- In formal writing, do not use symbols to replace words within a sentence.

  "Because Lemma 3 applies $\forall$ continuous functions, we can use it here."

  "Every function we have considered is differentiable, $\Rightarrow$ they are also continuous."

Organization.
In a novel, the element of surprise is key! In a mathematical paper, we do not want your audience to be surprised at all (except maybe when reading the abstract).

- Define terms as soon as they are introduced for the first time (or apologize).

  Not: The spider graph $G(n, k, 3)$ is a counterexample to that claim.

  Instead: The spider graph, defined below, is a counterexample to that claim.

- When defining a term, emphasize it.

  The graph that has n vertices and no edges is called the *null graph on n vertices*.

  Definition 4.1.3. The *chromatic number* of a graph G, denoted $\chi(G)$, is the least positive integer k for which there exists a proper vertex colouring of G using k colours.

- Introduce each theorem or lemma with at least a sentence of explanation.

  "The following lemma will help us to prove Theorem 3."

  "The following corollary follows immediately from Theorem 3 in the case where G is a path."

- Warn your reader if you make a statement that is not immediately obvious, but which you will justify later, or which is intended to lead to a contradiction.

  Not: Every triangulation has a vertex of degree at most five, and so it is easy to see by an inductive argument that G can be properly 6-coloured.

  Instead: We will demonstrate that every triangulation has a vertex of degree at most five, and so it is easy to see by an inductive argument that G can be properly 6-coloured.